HW 9 solutions Analog Filters

Ans. 2

(a) Circuit 1

Series RLC

\[ \omega_0 = \frac{1}{\sqrt{LC}} = 9 \times 2.5 \times 10^9 \]

\[ C = 1 \mu F \]

\[ L = 4 \mu H \]

\[ B\omega_0 = \frac{R}{L} \]

(b) Circuit 2

Parallel RLC

\[ L = 4 \mu H \]

\[ B\omega_0 = \frac{1}{\frac{1}{RC}} \]

(1) \[ R = 137.2 \Omega \]

(2) \[ R = 318.3 \Omega \]

(2) At 25 kHz i.e. at resonance frequency

\[ L, C \text{ cancel each other} \]

\[ \therefore \text{for Circuit 1 (Series RLC)} \]

\[ C_S = \frac{1}{\omega_0} = \frac{1}{\frac{1}{\frac{1}{2\pi 25 \times 10^3}}} = 2.5 \mu F \]

\[ \text{for Circuit 2 (Parallel RLC)} \]

\[ C_S = \frac{1}{\omega_0} = 0 \mu F \]

\[ (C) L, C \text{ cancel each other and effectively have an open circuit at op} \]

(c) Circuit 1

\[ B\omega_0 = \frac{(R + R_S)}{L} \]

\[ \frac{(R + R_S)}{L} = 11 \text{ } B\omega_0 \quad \text{(where } B\omega_0 \text{ is } \text{frequency cutoff)} \]
\[
\frac{R + R_3}{L} = 1.1 \times B_{w_0} = 1.1 \left( \frac{B_L}{L} \right)
\]

2) \[ \frac{R_3}{L} = 0.1 \left( \frac{B_L}{L} \right) \]

\[ R_3 = \frac{R_L}{10} = 1.27 \Omega \]

\[
\frac{1}{c (R + R_3)} = 0.9 \frac{v_R}{c R} \quad \Rightarrow \quad R + R_3 = 10 \frac{R}{9}
\]

\[ R_3 = \frac{R_L}{9} = 35.3 \Omega \]

\[ A_{m3} \quad (a) \quad \omega_B = \frac{1}{R_L} \]

b) for a low pass (secondary) chopper \[ B_{w_1} \approx 0 \quad \text{at} \]

moreover, it is clear that we require

maximally flat response, or butterworth response.

\[ a = \frac{1}{\sqrt{2}} \]

For series RLC \[ a = \frac{1}{2} \sqrt{\frac{1}{2}} = \frac{1}{2} \]

\[ D_L = \frac{R^2 C}{2} \]
For second order Butterworth filter

\[ B\Omega = \omega_0 = \frac{1}{\sqrt{2}} \]

in terms of \( R, C \) is \( \frac{1}{\sqrt{R C}} = \frac{1}{R C} \)

\[ H(s) = \frac{1}{(1 + \frac{s}{\omega_p})^2} \]

**Clk 2**

\[ V_o = A(v_i - v_0) \]

\[ V_o + A V_o = A v_i \]

\[ V_o(s) = \frac{A}{1 + A} \]

where \( A = \frac{1}{(1 + \frac{s}{\omega_p})^2} \)
\[ H(s) = \frac{1}{1 + \left( \frac{s}{\omega_p} \right)^2} = \frac{1}{1 + (1 + \frac{s}{j\omega_p})^2} \]

(b) 
\[ H(s) = \frac{1}{1 + \omega_0^2 + \frac{s^2}{\omega_p^2}} \]
\[ \omega_0 = \omega_p \quad \alpha = \frac{\omega}{\omega_p} \]
\[ = \frac{1}{\frac{\omega}{\omega_p} + \frac{s^2}{\omega_p^2}} \]
\[ \omega_0 = \sqrt{\omega_p} \quad \alpha = \frac{1}{\sqrt{2}} \]

\[ H(\omega) \]

\[ 0 \]

\[ -10 \]

\[ -180^\circ \quad \omega_p \quad \sqrt{3}\omega_p \quad 90^\circ \]
Problem 4

\[ \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{1 + \frac{1}{\omega C}} + \left(\frac{R}{\omega L} + R\right)} \]

\[= \frac{R}{\frac{1}{1 + \frac{1}{\omega C}} + \frac{R}{\omega L} + R\tau s L} \]

\[\omega^2 = \frac{\frac{R}{C}}{\frac{R}{L} + \frac{1}{C}} \]

\[\begin{align*}
\omega\sigma &= \frac{1}{\omega C} + \frac{R}{\omega L} \\
\alpha &= \frac{\omega_0}{\omega} \\
\frac{\partial \sigma}{\partial R} &= \omega_0 \frac{\partial}{\partial R} \left(\frac{1}{\omega C} + \frac{R}{\omega L}\right) \\
&= \omega_0 \left(-\frac{1}{\omega^2 C} + \frac{1}{\omega L}\right) \\
\frac{\partial \sigma}{\partial R} &= 0 \\
\implies R &= \sqrt{\frac{1}{L}} \\
\text{Max } \alpha &= \frac{1}{\sqrt{\omega}} \\
\alpha \omega_0 &= \text{constant} = \sqrt{\frac{\omega}{2L}}
\end{align*} \]