1. Initially, assume $R_S = 0$, $R_L = \infty$. Fig. 1 shows a first order filter whose input is the sum of two sinusoids $v_s(t) = 1V \cos(1 \text{ Mrad/s} t) + 1V \cos(1000 \text{ Mrad/s} t)$. The higher frequency sinusoid should be attenuated by 40 dB and the lower frequency sinusoid should be attenuated as little as possible.

(2 pts.) Determine the transfer function of the filter. Draw the schematic of a passive RC filter with $R = 100 \text{ k\Omega}$ that will accomplish this. What is the attenuation (in dB) of the lower frequency sinusoid?

(3 pts.) In the previously designed filter, if $R$ and $C$ can have variations of $\pm 10\%$, (a) What are the maximum and minimum values of the pole frequency? What is the percentage variation from the nominal value? (b) What is the worst case (smallest) attenuation of the higher frequency signal? (c) What is the worst case (largest) attenuation of the lower frequency signal?

(1 pt.) Reevaluate the transfer function with $R_S = 10 \text{ k\Omega}$, $R_L = 1 \text{ M\Omega}$. How would you restore the pole to the original value?

(1 pt.) With $R_S = 10 \text{ k\Omega}$, $R_L = 1 \text{ M\Omega}$, choose $R$, $C$ such that the pole of the filter is the same as originally determined. What is the transfer function? Determine the attenuation of the two sinusoids.

(3 pts.) Fig. 2 shows a cascade of $N$ identical buffered first order filter sections. $v_s(t) = 1V \cos(1 \text{ Mrad/s} t) + 1V \cos(10 \text{ Mrad/s} t)$. Using simple Bode Plots, determine the smallest $N$ required to reduce the higher frequency signal by 80 dB while leaving the lower frequency signal unchanged. What is the value of the pole of each section? Now, using the transfer function of the filter so obtained, find the actual attenuation of the two signals. Recompute $N$ and the pole of the filter if the lower frequency should be attenuated by $\leq 3$ dB and the higher frequency by $\geq 80$ dB.

3. (3 pts.) Design an ac coupling stage between the
two stages shown in Fig. 3. The second stage has an input capacitance $C_i = 1\, \text{pF}$. a) The attenuation for very high frequencies ($\omega \rightarrow \infty$) should be less than 1 dB, b) The attenuation for 10 Mrad/s should be less than 4 dB, c) The capacitor used in the circuit should be minimized. d) The dc bias provided to the 2nd stage should be 1 V (A 1 V dc source is available to you.).

4. (1 pt.) Design a filter with the transfer function $-k/(1 + s/p_1)$, with $k = 10$, $p_1 = 10$ Mrad/s. Draw the schematic with ideal opamps—use $C = 1\, \text{pF}$.

(2 pts.) Determine the dc gain $A_o$ and the unity gain frequency $\omega_u$ of the opamp such that each of these nonidealities (acting by itself) changes the pole of the filter by less than 2.5%.

(2 pts.) Draw the Bode plot of the loop gain for the filter you designed. Use an integrator model for the opamp with $\omega_u$ determined previously.

(2 pts.) Redesign the filter (use ideal opamps) assuming that the largest resistor allowed is 10 k$\Omega$. 

Figure 3: