Columbia University
Department of Electrical Engineering
Final Examination
EE 3106. Fall, 1995
December 15, 1995
Time: 3 hours
Total: 200 points

NAME

Read carefully before you proceed:

1. This exam is closed book, closed notes.

2. Put all answers in the space provided in the exam booklet.

3. Partial credit will be given. If you don’t have time to work a problem completely, explain how you would solve it and try to set up the problem. No credit will be given for answers without explanation.

4. Equations and constants are given on the last page. Please use them.
1. The following short-answer questions require only few sentence answers.

(a) Define avalanche breakdown as it applies to pn junction diodes. (6 points)
(b) The following is a semi-log plot of the current through a \textit{pn} junction as a function of bias. What temperature was this measurement taken? What are the physical origins of the slopes shown in the graph? (8 points)
(c) In terms of the lattice constant $a$ defined by the conventional unit cell, what is the nearest neighbor distance in an fcc crystal? (6 points)

(d) Why is the emitter of a bipolar junction transistor always more heavily doped than the base? (6 points)
(e) Define the base transit time in a bipolar junction transistor (both in words and mathematically) (8 points)

(f) What is channel length modulation in an FET? (6 points)
2. Consider the very special $p^+n$ diode shown below. The diffusion coefficient for minority carrier electrons on the n-side is given by $D_p$. Assume that on the n-side, there are no R-G centers for $0 \leq x \leq L$ (i.e., the minority carrier lifetime is infinite). Elsewhere, the diffusion length for holes is $L_p$. 

![Diagram of a p^+n diode with labels $V_A$, $p^+$, no R-G, and n. The length is denoted by $L$. The diagram shows the separation of the regions and the voltage applied.]
(a) Show that the minority carrier hole concentration on the n-side is given by: for $0 \leq x \leq L$,

$$p_n(x) = p_{no} e^{qV_A/kT} - \frac{p_{no}(e^{qV_A/kT} - 1)}{L + L_p} x$$

for $x \geq L$,

$$p_n(x) = p_{no} e^{qV_A/kT} \frac{L_p}{L + L_p} e^{-(x-L)/L_p}$$

Hint: $p_n$ and $dp_n/dx$ are both continuous at $x = L$. (15 points)
(b) What is the current density flowing through the diode? (8 points)
3. Consider the diode structure shown below. The junction breaks down when the maximum electric field in the junction reaches $E_{\text{crit}} = -2 \times 10^6 \text{V/cm}$ under a reverse bias voltage of $V_{BR}$.

(a) Show that when the maximum field in the junction is $-2 \times 10^6 \text{V/cm}$, the depletion region extends across the entire width $W_{\text{epi}} = 2 \mu\text{m}$. Sketch the electric field as a function of $x$. (10 points)
(b) Show that the breakdown voltage is given by the following expression:

\[ V_{BR} \approx \varepsilon_{cr} W_{epi} + \frac{q N_D W_{epi}^2}{2 \kappa \varepsilon_0} \]

Evaluate this expression to find the breakdown voltage for this diode. (10 points)
4. Consider an $n^+p^+\,\!$ bipolar junction transistor with an emitter injection efficiency ($\gamma$) of 1. $A = 10^{-4} \text{cm}^2$, $D_B = 20 \text{cm}^2/\text{sec}$, $\tau_B = 10^{-6} \text{sec}$, $N_B = 10^{16} \text{cm}^{-3}$. The width of the neutral base is $W_B = 4 \mu\text{m}$ (assumed to be independent of biasing). Assume that this transistor is used in the following circuit.

(a) Is the transistor in the saturation or forward active region of operation? What is the collector current $I_c$? $V_{CE}$? (12 points)
(b) Calculate the base transport factor \( (\alpha_T) \), the common base current gain \( (\alpha) \), and the common emitter current gain \( (\beta) \). (8 points)

(c) Find the small-signal equivalent circuit of this transistor at the operating point determined by the bias circuit. You may ignore the junction capacitances and include only \( g_m \), \( r_\pi \), and \( C_\pi \) in your model. (12 points)
(d) Find the magnitude of the small-signal frequency-dependent common-emitter current gain $\beta = |i_c/i_b|$ and find $f_T$, the frequency at which this current gain drops to 1. (14 points)
5. Suppose that the transistor of Problem #5 is used in the following circuit:
(a) At $t = 0$, the switch is flipped. How long does it take for the transistor to reach the edge of saturation? Sketch $I_c$ as a function of time. (15 points)

(b) What is the total amount of minority carrier (electron) charge stored in the base after the switch has been closed for a long time? (6 points)
6. Consider the n-channel silicon MOS transistor shown below. The gate is $n^+$ polysilicon. The oxide thickness is $t_{\text{oxide}} = 30\text{nm}$. The substrate doping is $N_A = 10^{15}\text{cm}^{-3}$. The effective surface mobility in the hole inversion layer is $\mu_e = 400\text{cm}^2/\text{V} - \text{sec}$, $W = 10\mu\text{m}$, $L = 2\mu\text{m}$.

![Diagram of n-channel silicon MOS transistor]

(a) Assuming that there is no fixed charge ($Q_F = 0$), calculate the flat-band voltage ($V_{FB}$), the gate voltage that must be applied to make the bands flat. (8 points)
(b) What is the threshold voltage ($V_T$)? (10 points)
The device is used in the following circuit.
(c) Sketch the capacitance measured between the gate and substrate as a function of $V_G$ at a measurement frequency of $f = 100 MHz$. What is the minimum capacitance? At what voltage? Your curve should look like the “low-frequency” C-V plot for the MOS-capacitor. Why? (12 points)
(d) Sketch the band diagram in the $x$ direction at $V_G = 2V$. What is the voltage drop across the silicon and the voltage drop across the oxide? (10 points)
Suppose that the device is now used in the following circuit:

(e) What is the drain current $I_D$ and the drain voltage $V_D$? Is the device in the linear or saturated region of operation? Sketch $I_D$ versus $V_D$. Sketch the load-line in the same plot and show the operating point calculated above. (10 points)
Equations and constants

\[ kT = 0.026 \text{ eV at } 300 \text{ K} \]

\[ N_v = 1.0 \times 10^{19} \text{ cm}^{-3} \text{ for Si at } 300 \text{ K} \]

\[ N_c = 2.8 \times 10^{19} \text{ cm}^{-3} \text{ for Si at } 300 \text{ K} \]

\[ n_s = 1.18 \times 10^{10} \text{ cm}^{-3} \text{ for Si at } 300 \text{ K} \]

\[ E_g = 1.12eV \text{ for Si at } T = 300 \text{ K} \]

\[ kT/q = 0.026V \text{ at } T = 300 \text{ K} \]

\[ \kappa_s = 11.7 \]

\[ \kappa_o = 3.9 \]

\[ \chi_{si} = 4.1eV \]

\[ \epsilon_o = 8.85 \times 10^{-14} \text{ F/cm} \]

\[ q = 1.6 \times 10^{-19} C \]

\[ g_m = \frac{qL}{kT} \]

\[ r_s = \frac{\beta_o}{g_m} \]

\[ C_s = g_m r_s \]

\[ \alpha_T = 1 - \frac{1}{2} \left( \frac{W}{L_m} \right)^2 \]

\[ V_T - V_{FB} = 2\Phi_F + \sqrt{2qk_s\epsilon_o N_A(2\Phi_F)/C_o} \]

\[ I_D = \frac{W}{L} \mu_n C_o(V_G - V_{TH} - V_D/2)V_D \]