Effects of Switching Pairs On CMOS Differential LC Tank Oscillators

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Abstract—This paper presents a detailed large signal analysis for the expression for CMOS differential LC tank oscillator. The approach starts from the impulse sensitivity function with the assumption of cyclostationary noise and large signal operation for the switching pair. The analysis leads to a solution which agrees with the result derived from the sideband mixing approach.

Index Terms—ISF, phase noise, switching pair, LC oscillator

I. INTRODUCTION

EARLY DAYS, while circuit simulators are not equipped with a specialized feature that is capable to compute the phase noise with well-controlled accuracy, oscillators were designed on the basis on empirical equations with less efficiency and were mostly unpredictable. Leeson [1] used linear feedback system to explain the phase noise, and however, his hypothesis mainly depends on an empirical noise factor, which does not give any information from the circuit itself. Then Hajimiri [2] proposed an approach based on the assumption of linear time varying system and a noise-to-phase response named impulse sensitivity function (ISF) with the same period of the oscillation. Actually Hajimiri’s approach is not only focused on differential LC oscillators but also for the kinds without a tank resonator. Even though there is no example given for the oscillator with cyclostationary noise sources in his paper, he proposed that the ISF could be combined with the cyclostationarity of the noise source by simple multiplication. Rather than giving a clear idea of how exactly the switching pair, Hajimiri mainly relies on the simulator to find the ISF for the switching pair. In [3], Ham follows the ISF approach to give the phase noise caused by the switching pair to optimize the sizing while cyclostationarity is not assumed.

Later Rael and Abidi [4] proposed a closed form expression for the phase noise of differential LC oscillators. The analysis is based on the similarity of switching pair in both oscillators and mixers [5]. For the switching pair, simple hard switching is assumed for the I-V characteristic, and the result states that the sizing will not have any effect on the phase noise. In Demir’s proposal [6], he assumed the noise sources are only stationary in oscillators since oscillators are autonomous and not driven by any external periodic reference sources. This assumption can result in a Lorenzian spectral shape rather than a simple $1/f^2$ characteristic for a white noise source. Probably Demir’s approach is the most accurate one that reflect the real mechanism that govern the electrical noise to phase noise which is defined as the sideband power divided by the signal power.

In section II, the derivation of the noise factor caused by the switching pair is presented. In section III, numerical simulations are taken to verify these derivations. In section IV, the effect introduced by the switching pair are examined and discussed with the aid of circuit simulator, and the conclusion is made in the final section.

II. MATHEMATICAL DERIVATION

Here the noise contribution from the switching pair is presented through Impulse sensitivity function (ISF) approach. Four assumptions are made in the following derivation:

1. The RLC tank is not loaded by the active part, which means the quality factor Q remains the same over the entire oscillation cycle.
2. Linearity for electrical noise to phase noise conversion.
3. ISF for an unloaded RLC tank is a sinusoidal and is shifted by $\pi/2$ compared to the voltage waveform, which mean it reaches its maximum while the differential tank voltage is zero.
4. Transistors are square law devices

First we assume the topology is composed of a switching pair biased by a tail current source as depicted in Fig. 1. While the switching pair is driven differentially by a large sinusoidal signal, the transistors operate over the triode, saturation, and cut-off regions as illustrated in Fig. 2. In the cut-off region, the transistor does not contribute any noise. And while operating in the triode region, the transistor acts like a resistor. The equivalent circuit can be illustrated as Fig. 3.

If infinite and wideband common-mode rejection ratio is assumed which means there are no extra signal path in parallel with the current source, such as parasitic capacitance or conductance, the noise current can only circulate in the turn-on resistance introduced by the transistor in the triode region because there is no return path through the tail current source.
Therefore the noise mainly originates from the saturation region. From the previous assumption that both switching transistors are in saturation region and that the transistors are square law devices, the I-V characteristic is given as follows:

\[
I_{\text{out}}(V_d) = \frac{I_{\text{tail}}}{2} + \frac{I_{\text{tail}} V_d}{V_{\text{OD}}} \cdot \sqrt{1 - \left(\frac{V_d}{2 V_{\text{OD}}}\right)^2}
\]  

(1)

And certain substitution is further made:

- \( I_0 = \frac{I_{\text{tail}}}{2} \)  
- \( V_{\text{OD}} = V_{\text{GS}} - V_T \)  
- \( gm = \frac{2 I_0}{V_{\text{GS}} - V_T} \)

where \( I_{\text{tail}} \) is the tail biasing current, \( V_d \) is the differential tank amplitude, and \( V_{\text{OD}} \) is the overdrive voltage. The I-V characteristic is also illustrated in Fig. 4.

By assuming equation (1) equals to zero, the threshold voltage at which the switching pair fully switches the tail current to one single branch can be found:

\[
\Delta V = \sqrt{2 V_{\text{OD}}}
\]  

(5)

For a single switching transistor driven by a large signal, the input-dependent large signal transconductance can be defined as the derivative of I-V characteristic:

\[
Gm(V_d) = \frac{\partial I}{\partial V_d}
\]  

(6)
Apply equation (6) in equation (1), the signal dependent transconductance is found:

\[ G_m(V_d) = \frac{I}{V_{od}} \left[ \frac{1}{2} \left( \frac{V_d}{V_{od}} \right)^2 + \frac{1}{2} \left( \frac{V_d}{V_{od}} \right)^2 \right] \]

(7)

Factorize equation (7), equation (8) can be obtained:

\[ G_m(V_d) = \frac{I}{2V_{od}} \left[ 1 - \frac{2V_d^2}{V_{od}^2} \right] \]

(8)

In order to facilitate the upcoming derivation, substitutions are further made:

\[ G_m(V_d) = G_m_0 \cdot \sqrt{1 - \frac{2V_d^2}{V_{od}^2}} \] \hspace{1cm} (9)

\[ G_m_0 = \frac{I_{s}}{2V_{od}} \] \hspace{1cm} (10)

Here we assume that the transconductance is driven by a periodic large signal, and \( V_d \) is replaced by the sinusoidal signal from the oscillation. The relationship between transconductance and oscillation phase can be constructed:

\[ G_m(\theta) = G_m_0 \cdot \sqrt{1 - \frac{2V_0 \sin(\theta)/V_{od}^2}{1 - (V_0 \sin(\theta)/V_{od})^2}} \] \hspace{1cm} (11)

where \( V_0 \) is the differential amplitude of the tank. However, strictly speaking, the phase in equation (11) cannot be simply treated as the oscillation cycle since the oscillation is autonomous and not driven externally. And the phase uncertainty will drift without bound and the transconductance lose its periodicity. If this erroneously assumed periodicity is adopted to obtain the noise cyclostationarity, the lorezian spectral property will be ignored. For long-term noise property, which is equivalent to closed-in phase noise, this assumption can cause a severe error, but in the short-term noise characteristic, where \( 1/f^2 \) roll-off is dominant, this assumption of periodicity will not give much difference from the real oscillation mechanism. Therefore the implicit assumption of cyclostationarity is still adopted since it can greatly reduce the complexity of analysis.

The Impulse sensitivity function (ISF) for a simple R-L-C tank based oscillator with amplitude regulation is assumed [3]:

\[ \Gamma_\theta(\theta) = \cos(\theta) \] \hspace{1cm} (12)

This assumption can be only used with care since the negative resistance in an oscillator is always a nonlinear component which regulates the amplitude so that a steady oscillation is maintained. However, since the noise from the switching pair will have effects on phase noise only when in saturation region. All the circuit devices behave as its small signal equivalent, and the ISF only has to be accurate in this region.

Based on previous assumptions, the noise from the transistors of the switching pair is a cyclostationary process, and the current noise power spectral density (PSD) is:

\[ 4kT \cdot \gamma \cdot G_m(V_d) \] \hspace{1cm} (13)

Again, this step implicitly assume the cyclostationarity of noise. In order to simplify the analysis, Hajimiri proposed to derive an effective ISF function for the noise source so that we can treat the noise source as a stationary noise. In other words, the effective ISF for the switching pair should include the periodic part of the transconductance in equation (11).

\[ \Gamma_{\theta, \text{eff}}(\theta) = \Gamma_\theta(\theta) \cdot \left[ 1 - \frac{2V_0 \sin(\theta)/V_{od}^2}{1 - (V_0 \sin(\theta)/V_{od})^2} \right]^{1/2} \] \hspace{1cm} (14)

Notice that the square root is needed because the ISF function is for voltage-to-phase response. With this assumption, we can assume the noise from the switching pair is stationary, and the single-side-band (SSB) noise PSD can be written as:

\[ \frac{i_n^2}{N} = 4kT \cdot \gamma \cdot G_m_0 \] \hspace{1cm} (15)

According to equation (5) while the differential tank voltage goes over than \( \Delta V = \sqrt{2} \cdot V_{od} \), the current will be fully switched to only one branch so that the transconductance becomes zero since the current cannot increase with further increase in tank voltage. So that the conducting angle can be found:

\[ V_0 \sin(\theta_c) = \Delta V = \sqrt{2} \cdot V_{od} \] \hspace{1cm} (16)

\[ \theta_c = \sin^{-1} \left( \sqrt{2} \cdot \frac{V_{od}}{V_0} \right) \] \hspace{1cm} (17)
And from equation (14), (16), and (17), the effective ISF power can be summarized as:

\[|\Gamma_{ef}^2(\theta)|^2 = |\Gamma_{ef}(\theta)|^2 \cdot \left(1 - 2 \left(\frac{V_o \cdot \sin(\theta)}{V_{OD}}\right)^2\right)\]

when\[-\theta_c \leq \theta \leq \theta_c\]

\[|\Gamma_{ef}^2(\theta)|^2 = 0, \text{ elsewhere}\]

(18)

(19)

Since the effective ISF function is obtained and the stationarity of the transistor noise can be assumed, the noise current from the switching pair is now a white and stationary noise source. The analysis above is summarized and illustrated in Fig. 5.

The noise portions around different harmonic frequencies have the conversion gain equals to the Fourier coefficients of the ISF function. Since the white noise source is made equivalent to a stationary one. The electrical noise to phase noise conversion will only see a noise source with a flat spectrum, which means the total converted noise power would be the power sum of the entire Fourier coefficients. According to the power conservation property introduced by Parseval’s theorem, the sum of the coefficients can be found by:

\[\sum_{n} c_n = \frac{1}{\pi} \int_{0}^{2\pi} |\Gamma_{ef}(\theta)|^2 \, d\theta = 2 \cdot \Gamma_{\text{rms}}^2\]

(20)

Applying (18) and (19) to (20),

\[2 \cdot \Gamma_{\text{rms}}^2 = \frac{1}{\pi} \int_{-\theta_c}^{\theta_c} \left|\Gamma_{ef}(\theta)\right|^2 \cdot \left(1 - 2 \left(\frac{V_o \cdot \sin(\theta)}{V_{OD}}\right)^2\right) \, d\theta\]

(21)

First we replace the ISF with (12)

\[2 \cdot \Gamma_{\text{rms}}^2 = \frac{1}{\pi} \left(1 - 2 \left(\frac{V_o \cdot \sin(\theta)/2}{V_{OD}}\right)^2\right) \, d\theta\]

(22)

To compute the integral, we first replace the sinusoidal with another variable \(\lambda = \sin(\theta)\):

\[d\theta = \frac{1}{\cos(\theta)} \cdot d\lambda = \frac{d\lambda}{\pm \sqrt{1 - \sin^2(\theta)}}\]

(24)

Since the integral only cover the conducting angle which is assumed to be within \(\pm \pi /2\), and therefore positive root should be chosen.

\[d\theta = \frac{d\lambda}{\sqrt{1 - \sin^2(\theta)}} = \frac{d\lambda}{\sqrt{1 - \lambda^2}}\]

(25)

In addition, the boundaries of the integral should also be adjusted. According to (16), the new boundaries can be found as:

\[\pm \lambda_c = \sin(\theta_c) = \pm \sqrt{\frac{2 V_{OD}}{V_o}}\]

(26)

Then equation (23) can be rewritten as:

\[2 \cdot \Gamma_{\text{rms}}^2 = \frac{1}{\pi} \left(1 - 2 \left(\frac{V_o \cdot \lambda/2}{V_{OD}}\right)^2\right) \, d\lambda\]

(27)

(28)
Equation (28) cannot have closed form solution, and the variables in the square roots should be made approximation. It is found that the first term in the integral is more suitable for approximation:

\[ 2 \cdot \Gamma_{m_0}^2 = \frac{1}{\pi} \left[ 2 \cdot \int_{-\infty}^{\infty} \left[ 1 - \frac{\lambda^2}{2} \right] \left[ 1 - 2 \cdot \frac{V_\omega \cdot \lambda / 2}{V_{OD}} \right]^2 \left[ 1 - \frac{V_\omega \cdot \lambda / 2}{V_{OD}} \right]^2 \right] d\lambda \]  

(29)

Applying (17) to (26), the final result is:

\[ 2 \cdot \Gamma_{m_0}^2 = \frac{1}{\pi} \left[ 2 \cdot \left( \frac{V_{OD}}{V_0} \right)^2 \left( 2 - \frac{\pi}{2} \right) \left( \frac{V_{OD}}{V_0} \right)^4 \right] \]  

(30)

\[ \Gamma_{m_0}^2 = \frac{1}{\pi} \left[ 2 \left( \frac{V_{OD}}{V_0} \right)^2 \left( 2 - \frac{\pi}{2} \right) \left( \frac{V_{OD}}{V_0} \right)^4 \right] \]  

(31)

For SSB representation, the expression in [2] for phase noise can be used:

\[ L(\Delta \omega) = 2 \cdot \frac{\Gamma_{m_0}^2}{4 \cdot q_{MAX}^2} \frac{\pi^2}{\Delta \omega^2} \]  

(32)

where \( q_{MAX} \) is the maximum charge displacement in an oscillation cycle.

\[ q_{MAX}^2 = \left( V_0 \cdot C \right)^2 \]  

(33)

And then equation (31) can be simplified as:

\[ L(\Delta \omega) = \frac{2 \cdot \frac{1}{q_{MAX}^2}}{4 \cdot \Delta \omega^2} \left[ \Gamma_{m_0}^2 \frac{\pi^2}{\Delta \omega^2} \right] \]  

(34)

\[ L(\Delta \omega) = \frac{2}{V_0^2 \cdot C^2} \frac{1}{4 \cdot \Delta \omega^2} \left[ \Gamma_{m_0}^2 \frac{\pi^2}{\Delta \omega^2} \right] \]  

(35)

\[ L(\Delta \omega) = \frac{2}{V_0^2 \cdot C^2} \frac{1}{4 \cdot \Delta \omega^2} \left[ \Gamma_{m_0}^2 \frac{\pi^2}{\Delta \omega^2} \right] \]  

(36)

Since quality factor can be expressed as:

\[ Q^2 = R_p^2 \cdot \frac{Q^2}{\Delta \omega^2} \cdot C^2 \]  

Equation (36) can be rewritten as:

\[ L(\Delta \omega) = \frac{2 \cdot R_p^2}{V_0^2} \cdot \frac{1}{4 \cdot \Delta \omega^2} \left[ \Gamma_{m_0}^2 \frac{\pi^2}{\Delta \omega^2} \right] \]  

(37)

After proper re-arrangement, equation (38) is rewritten as:

\[ L(\Delta \omega) = \frac{2 \cdot R_p^2}{V_0^2} \cdot \left[ \Gamma_{m_0}^2 \frac{\pi^2}{\Delta \omega^2} \right] \left( \frac{\omega_0}{2 \cdot Q \cdot \Delta \omega} \right)^2 \]  

(39)

Applying (10), (15), and (31) in (39),

\[ L(\Delta \omega) = \frac{4kT \cdot R_p}{V_0^2} \left( \frac{\omega_0}{2 \cdot Q \cdot \Delta \omega} \right)^2 \left[ 2 - \frac{\gamma \cdot I_{sat} \cdot R_p}{\pi \cdot V_0} \right] \left[ \frac{2 - \frac{\pi}{2} \cdot \frac{V_{OD}}{V_0} \left( V_{OD} \right)^2}{\frac{V_0}{V_{OD}} \times \frac{V_0}{V_{OD}}} \right] \]  

(40)

It is more clear to rewrite equation (40) as follows:

\[ L(\Delta \omega) = \frac{4kT \cdot R_p}{V_0^2} \left( \frac{\omega_0}{2 \cdot Q \cdot \Delta \omega} \right)^2 \left[ 2 - \frac{\gamma \cdot I_{sat} \cdot R_p}{\pi \cdot V_0} \right] \left[ 1 - \left( 1 - \frac{\pi}{4} \right) \left( \frac{V_{OD}}{V_0} \right)^2 \right] \]  

(41)

If the overdrive voltage is so small compared to the differential tank amplitude that it could be approximated to zero, then equation (41) can be re-written as:

\[ L(\Delta \omega) = \frac{4kT \cdot R_p}{V_0^2} \left( \frac{\omega_0}{2 \cdot Q \cdot \Delta \omega} \right)^2 \left[ 2 - \frac{\gamma \cdot I_{sat} \cdot R_p}{\pi \cdot V_0} \right] \left[ 1 - \left( 1 - \frac{\pi}{4} \right) \left( \frac{V_{OD}}{V_0} \right)^2 \right] \]  

(42)

which agreed with Rael and Abidi’s result while ISF approach proposed by Hajimiri has been taken. Since the I-V relation in saturation region can be formulated as:

\[ I_0 = \frac{I_{sat}}{2} = \frac{1}{2} \cdot \mu C_{ox} \cdot \frac{W}{L} \cdot V_{OD} \]  

(43)

\[ L(\Delta \omega) = \frac{4kT \cdot R_p}{V_0^2} \left( \frac{\omega_0}{2 \cdot Q \cdot \Delta \omega} \right)^2 \left[ 2 - \frac{\gamma \cdot I_{sat} \cdot R_p}{\pi \cdot V_0} \right] \left[ 1 - \left( 1 - \frac{\pi}{4} \right) \left( \frac{V_{OD}}{V_0} \right)^2 \right] \]  

(44)

Therefore the aspect ratio of the transistor will have only a secondary effect on phase noise while the oscillator is current biased. If even higher order approximation for the first term in (28) in adopted,

\[ \sqrt{1 - \lambda^2} \approx 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} \]  

(45)

Following the similar procedure, equations (40) will become:

\[ L(\Delta \omega) = \frac{4kT \cdot R_p}{V_0^2} \left( \frac{\omega_0}{2 \cdot Q \cdot \Delta \omega} \right)^2 \left[ 2 - \frac{\gamma \cdot I_{sat} \cdot R_p}{\pi \cdot V_0} \right] \left[ 1 - \left( 1 - \frac{\pi}{4} \right) \left( \frac{V_{OD}}{V_0} \right)^2 \right] \]  

(46)

Again equation (46) can be interpreted in terms of aspect ratio through equation (42). If the operation is in current limited case, then:
\[ I_{\text{sat}} \cdot R_p = \frac{\pi}{2} \cdot V_0 \tag{47} \]

Applying equation (47), equation (46) becomes:
\[ L(\Delta \omega) = \frac{4kT}{V_e} \left( \frac{\alpha_0}{2Q \cdot \Delta \omega} \right)^2 \left[ \left( -1 - \left( \frac{Q}{4} \right) \left( \frac{V_0}{V_e} \right) \right) - \left( \frac{5}{6} \right) \left( \frac{V_0}{V_e} \right)^2 \right] \tag{48} \]

If we take a close look at both equation (41) and (48), the term containing the over drive voltage is reflecting the conducting angle. It is concluded that the larger the conducting angle is, the less phase noise coming from the switching pair while keep the biasing current and the tank property the same. However, it should be noticed that this conclusion could be only valid if the intrinsic ISF is a sinusoidal. If it is not, this dependence would be different, and the discussion is listed in the appendix. In real cases, the intrinsic is usually not a simple sinusoidal function, and therefore it results in different trends for optimizations on the transistor sizing. However, phase noise has low sensitivity on these higher order effects.

### III. NUMERICAL VERIFICATION

In order to verify this closed form result, SpectreRF is used to verify the derivation. In the simulation, only the thermal noise from the switching pair is considered. Some of the necessary parameters for the hand calculation are extracted from the model itself through simple DC/AC simulation. They are explained as follows:

1. \( \omega_0 \) is taken from the PSS simulation, not from \( 1/\sqrt{LC} \).
2. Quality factor \( Q \) is computed through equation (37).
3. \( \alpha_0 \) is obtained through AC simulation and with the same biasing in oscillator.
4. \( \gamma \) is computed from equation (15) by applying noise current obtained from AC noise simulation
5. \( V_0 \) is taken from PSS simulation

Since the most parameters are accurate enough, it proves that this closed form solution properly capture this noise sideband generation process. Even though it seems for accurate hand calculation many parameters should still be available from the simulation. However, if certain simplification is made for pure hand calculation purpose, acceptable error level can still be guaranteed. The alternative parameter can be summarized as:

\[ \omega_0 = \frac{1}{2 \pi \cdot \sqrt{L_p \cdot C_p}} \tag{49} \]

\[ Q = R_p \cdot \omega_0 \cdot C_p \tag{50} \]

\[ \frac{\alpha_0}{2Q \cdot \Delta \omega} = \frac{1}{2 \cdot R_p \cdot C_p \cdot \Delta \omega} \tag{51} \]

4. \( \gamma = 3/4 \), which is strongly dependent on the short channel effect.
5. \( V_{\text{OD}} = 200 \text{mV} \) at which the transistors are biased in strong inversion.
6. \( V_0 = \frac{2}{\pi} \cdot I_{\text{sat}} \cdot R_p \) at which the oscillator is assumed in current limited region

From equation (51), it turns out that the oscillation frequency will not make any difference whether from the simulation or equation (49). Finally, equation (52) can be used for fast calculation without any aid of numerical simulation.

\[ L(\Delta \omega) = \frac{4kT}{V_e} \left( \frac{1}{2 \cdot C_p \cdot \Delta \omega} \right)^2 \left[ \left( -1 - \left( \frac{Q}{4} \right) \left( \frac{V_0}{V_e} \right) \right) - \left( \frac{5}{6} \right) \left( \frac{V_0}{V_e} \right)^2 \right] \tag{52} \]

If the above parameters are applied to equation (52), the phase noise is computed \(-137.15 \text{dBc} \), which is 0.085dB from the simulated result. All the parameters used in the simulation comparison are listed in Table 1.

### IV. CONCLUSION

A large signal phase noise analysis on the basis of effective ISF is presented to find out the noise contribution of the switching pair in a CMOS differential LC oscillator. In this analysis, we made several assumptions, which might be invalid in some real cases. In the appendix, we assume an ISF that is 1 in the saturation region and zero in triode and cut-off region, a different trend for the over drive voltage (or equivalently,
processes are taken to verify this dependency on the intrinsic ISF cannot be determined. Similar simulations with different relationship between the overdrive voltage and the phase noise, therefore without the detailed intrinsic ISF, the exact sizing) is shown. However, since the coefficients for higher order terms are relatively small compared to one, the overdrive should not have a strong effect in most cases.

**APPENDIX**

In order to show how the ISF would have effects on the relationship between the overdrive voltage and the phase noise, we first assume the extreme case where ISF=1 while the switching pair generates noise. And equation (22) would become:

\[
2 \cdot \Gamma_{rms}^2 = \frac{1}{\pi} \left\{ 2 \cdot \int_{-\theta}^{\theta} \left[ \frac{1 - 2 \left( \frac{V_0 \cdot \sin(\theta)/2}{V_{OD}} \right)^2}{\sqrt{1 - \left( \frac{V_0 \cdot \sin(\theta)/2}{V_{OD}} \right)^2}} \right] d\theta \right\}
\]

(53)

And if we follow the same variable substitution procedure to derive the effective ISF, equation (27) would become:

\[
2 \cdot \Gamma_{rms}^2 = \frac{1}{\pi} \left\{ 2 \cdot \int_{-\lambda}^{\lambda} \left[ \frac{1 - 2 \left( \frac{V_0 \cdot \lambda/2}{V_{OD}} \right)^2}{\sqrt{1 - \left( \frac{V_0 \cdot \lambda/2}{V_{OD}} \right)^2}} \right] \cdot \frac{1}{\sqrt{1 - \lambda^2}} d\lambda \right\}
\]

(54)

And equation (31) becomes:

\[
\Gamma_{rms}^2 = \frac{1}{\pi} \left\{ 2 \cdot \left( \frac{V_{OD}}{V_0} \right) + (2 - \frac{\pi}{2}) \left( \frac{V_{OD}}{V_0} \right)^3 \right\}
\]

(55)

And the phase noise expression becomes:

\[
L(\Delta \omega) = \frac{4kT R_e}{V_0^2} \left\{ \frac{\omega_h}{2 \cdot \Delta \omega} \right\}^3 \left[ \frac{2 \cdot r \cdot I_{OD} \cdot R_e}{\pi \cdot V_s} \right] \left[ 1 + \left( \frac{V_{OD}}{4 \cdot V_0} \right) \right]
\]

(56)

With even more detailed analysis, equation (46) becomes:

\[
L(\Delta \omega) = \frac{4kT R_e}{V_0^2} \left\{ \frac{\omega_h}{2 \cdot \Delta \omega} \right\}^3 \left[ \frac{2 \cdot r \cdot I_{OD} \cdot R_e}{\pi \cdot V_s} \right] \left[ 1 + \left( \frac{V_{OD}}{4 \cdot V_0} \right) \right] \left[ \frac{3 \pi^2}{2} \right] \left( \frac{V_{OD}}{V_0} \right)^3
\]

Therefore without the detailed intrinsic ISF, the exact relationship between the overdrive voltage and the phase noise cannot be determined. Similar simulations with different processes are taken to verify this dependency on the intrinsic ISF.