Analysis of Noise and Interference in Companding Signal Processors

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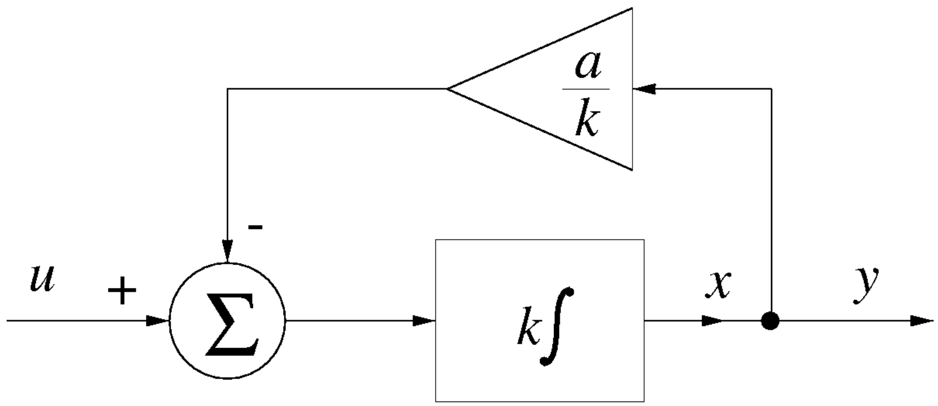
Outline

- Companding filters.
- Analysis of response to noise.
- Comparison with noise in purely linear filters.
- Examples and experimental results.
- Conclusions.

Motivation

- Companding (log domain and other types) filters are expected to result in a larger dynamic range for the same power.
- Companding filters use signal compression at the input, expansion at the output.
- Externally linear, but non-linear from the internal points to the output.
- Behavior of noise is different from that in conventional linear filters.

Linear first-order filter prototype



$$\dot{x} = -ax + ku$$
$$y = x$$

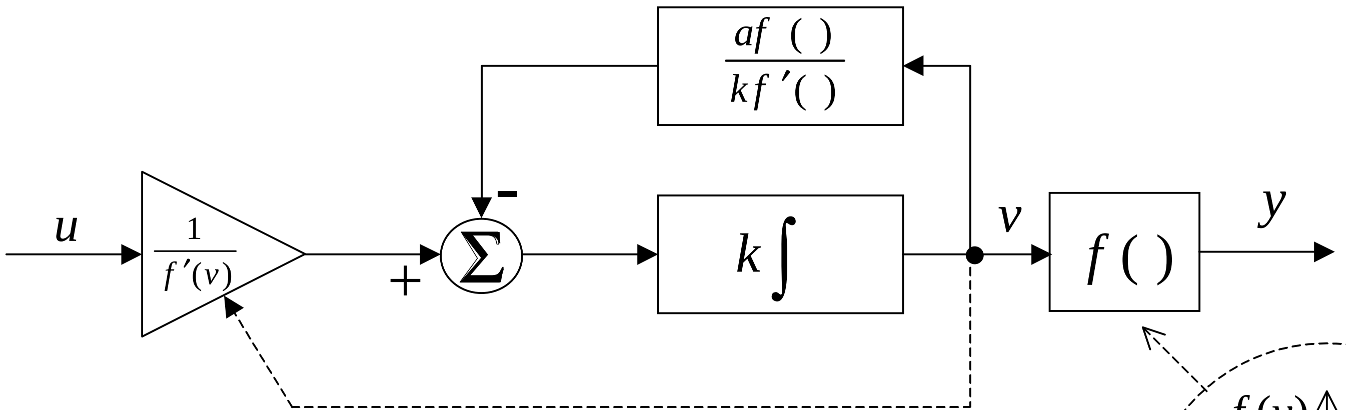
x : state variable

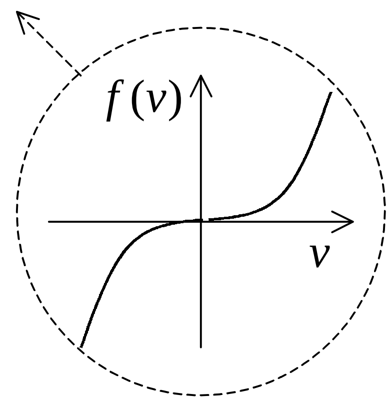
y : output

k, a : constants

$$H(s) = \frac{Y(s)}{U(s)} = \frac{k}{s+a}$$

Externally linear, internally non-linear first-order filter





$$\dot{v} = -a\,\frac{f(v)}{f'(v)} + \frac{k}{f'(v)}\,u$$

$$y = f(v)$$

$$\frac{Y(s)}{U(s)} = \frac{k}{s+a}$$

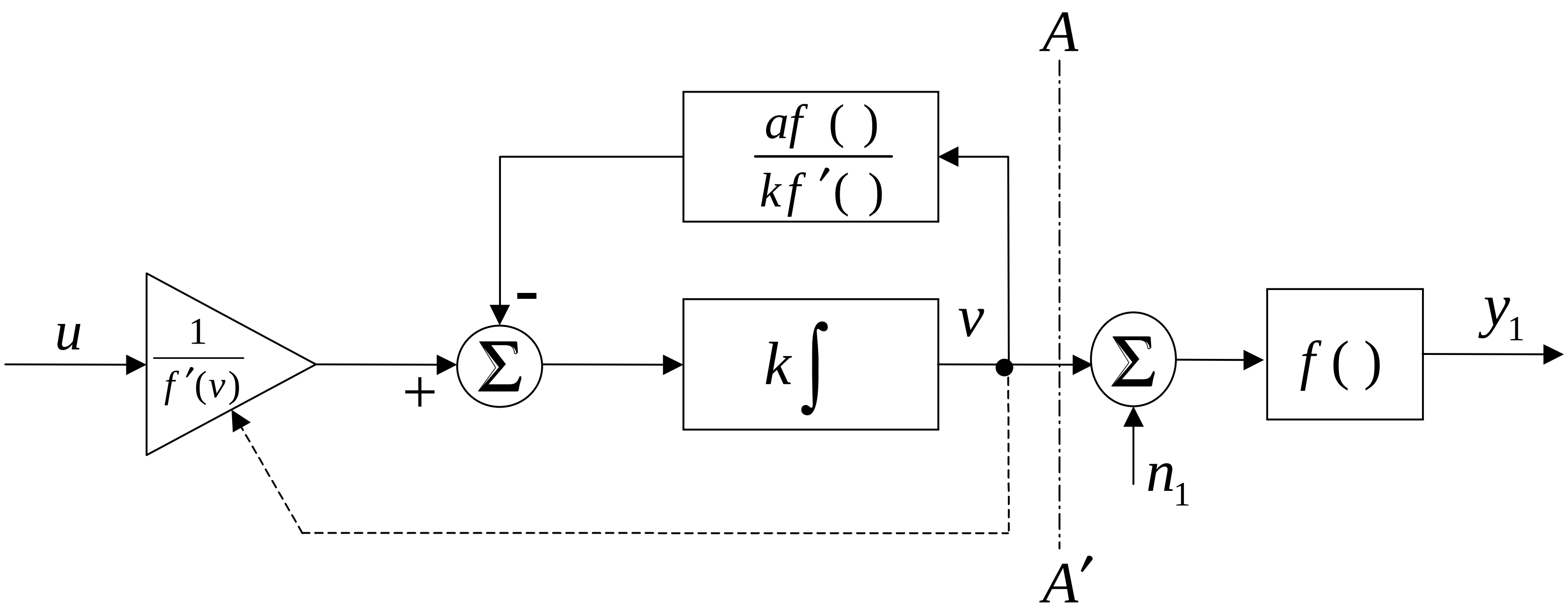
- v : new state variable related to x: $x = f(v)$
- Externally equivalent to the linear prototype.

Externally linear, internally non-linear first-order filter

- Nonlinear blocks at the input and the output.
- Internal noise is processed by signal-dependent transfer functions.
- Noise can be added at various points.

∴ Classical techniques for noise analysis of LTI systems are not applicable.

1. Noise added at the input of the expandor



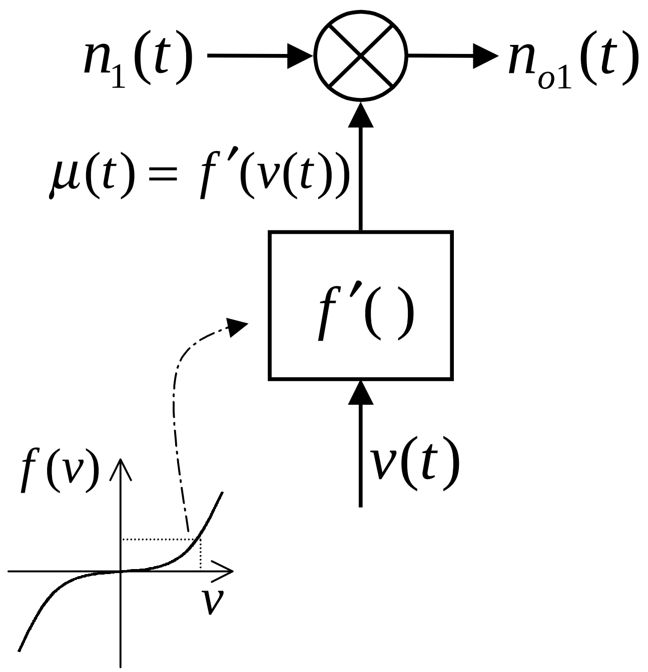
$$y_1 = f(v + n_1)$$

$$y_1 \approx f(v) + f'(v)n_1$$

$$n_{o1}(t) = y_1(t) - y(t) = \mu(t)n_1(t)$$

$$\mu(t) = f'(v(t))$$

Noise equivalent system



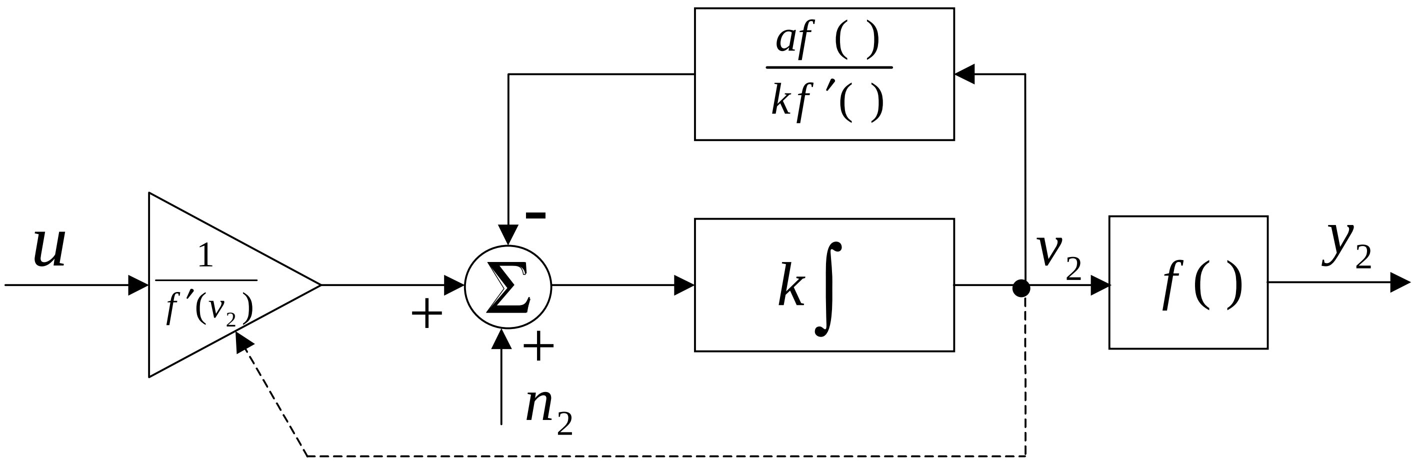
- Linear time varying (LTV) system.
- $f'(v)$: “gain” of the non-linear block f at the operating point “v”.
- Response to a delayed impulse $\delta(t - \tau)$:

$$h_1(t,\tau) = \mu(\tau)\delta(t-\tau)$$

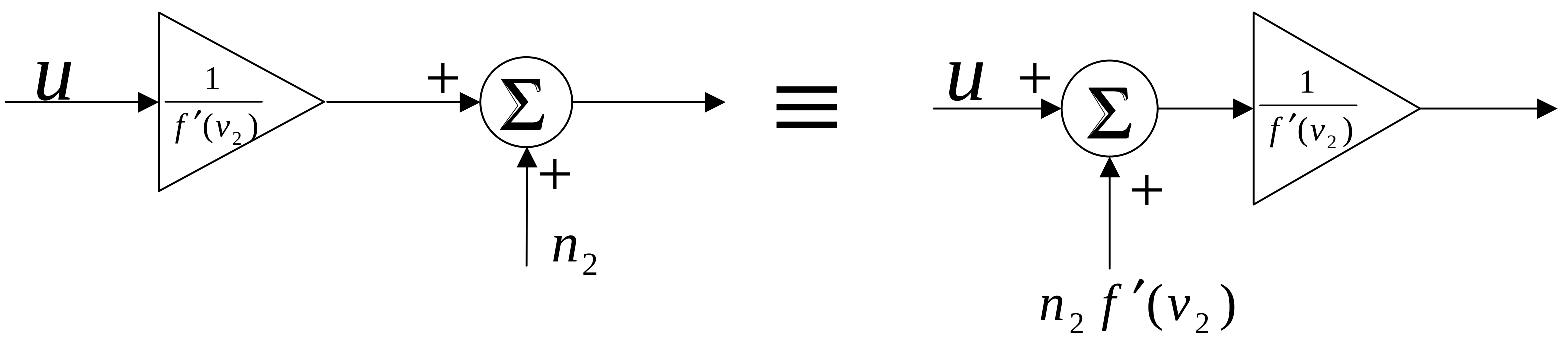
- If noise n_1 is stationary and input u is periodic: (Rice-‘70)

$$S_{o1}(\omega) = S_1 \frac{1}{T}\int_0^T \mu^2(\tau)d\tau$$

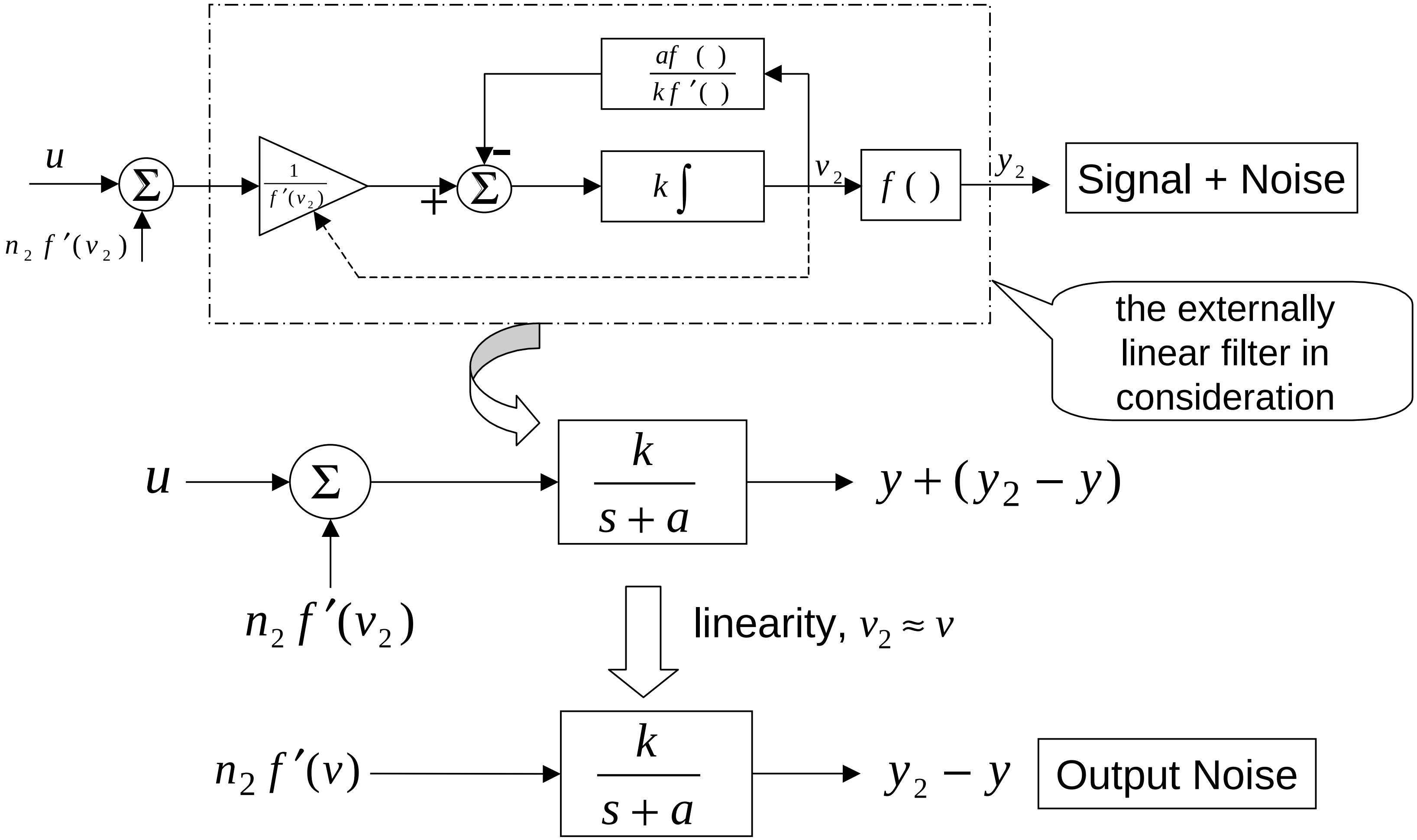
2. Noise added at the input of the integrator



v_2: modified state variable.



Noise equivalent system



∴ Output noise = modulated, filtered input noise.

Output noise power spectral density

- Stationary white noise n_2 (PSD = S_2), periodic input signal u:

$$S_{o2}(\omega) = S_2 \cdot \eta \, | H(\omega) |^2$$

- Weighting factor η:

$$\eta = \frac{1}{T} \int_0^T \mu^2(\tau) d\tau$$

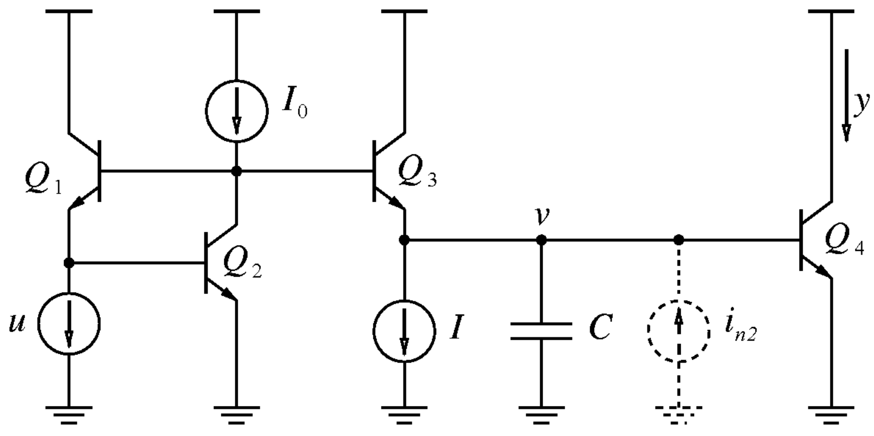
- For the first order low pass filter example:

$$S_{o2}(\omega) = \eta \frac{k^2}{a^2 + \omega^2} S_2$$

Behavior of noise in companding filters

- Output noise is *modulated by the signal*.
- Output noise depends on the *strength* of the signal.
- Output noise depends on the *shape* of the signal.
- In a high order filter, a large *out of band* signal at intermediate points can cause a large output noise *inside the desired band*.

First-order class A log-domain filter



$$f(v) = I_s \exp\left(\frac{v}{V_t}\right)$$

$$n_2 = \frac{V_t}{I_0} i_{n2}$$

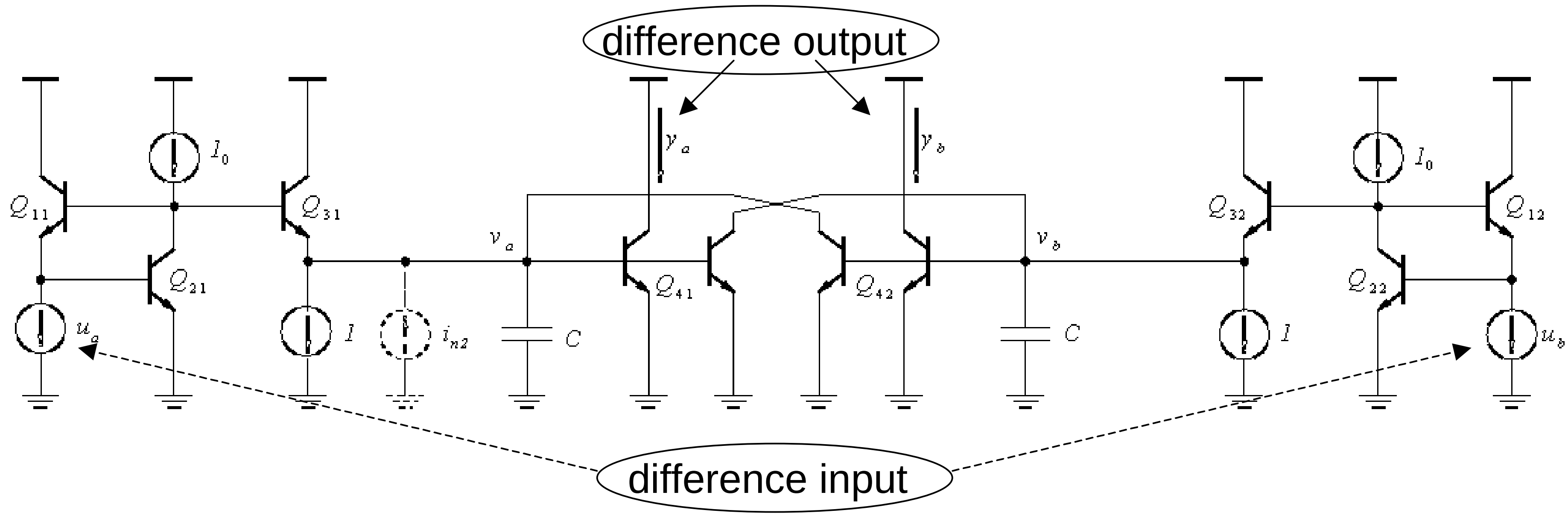
$$u(t) = I_{dc1} + I_{s1}\cos(\omega t)$$

$$y(t) = I_{dc2} + I_{s2}\cos(\omega t + \varphi)$$

$$\eta = \frac{1}{V_t^2}\left(I_{cy}^2 + \frac{1}{2} I_{py}^2\right)$$

- $I_{py} \le I_{cy} \rightarrow \eta$ varies by 1.8 dB (not much!)

First order class B log-domain filter

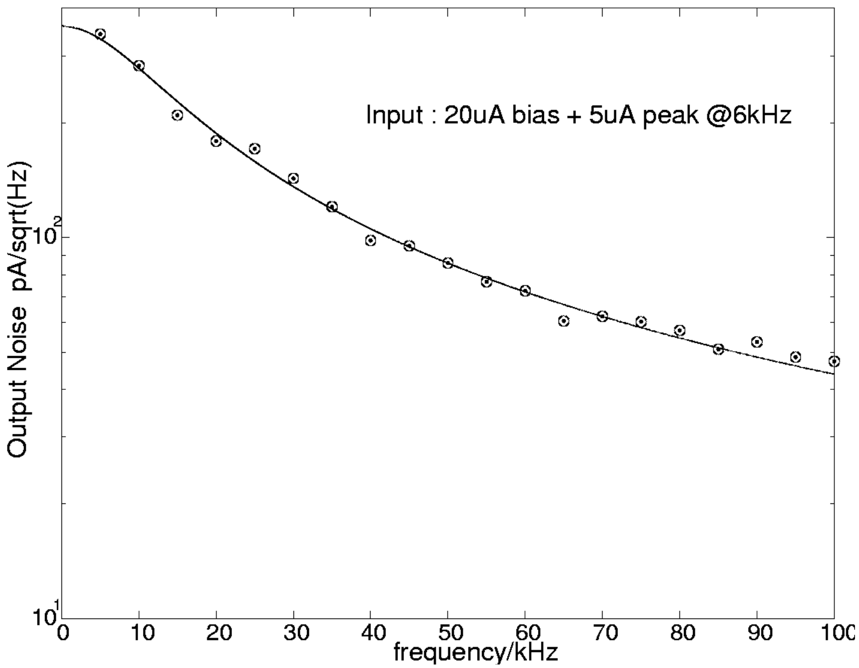


$$\mu(t) = f'(v_a(t)) = \frac{1}{V_t} y_a(t)$$

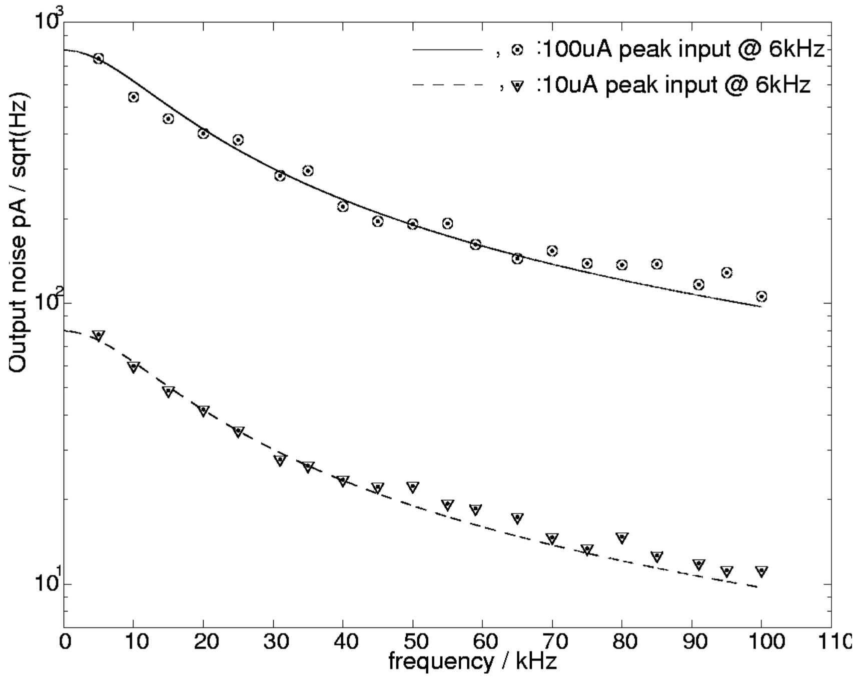
- Each side is on for approximately half a cycle of the input.
- For a sinusoidal input, $\eta = \dfrac{I_{py}^2}{4V_t^2}$

Measured results

Class A



Class B



$k = a =$ 12.3 krad/s

i_{n2} = 350 pA / sqrt(Hz)

Conclusions

- The behavior of noise in an instantaneously companding filter has been analyzed.
- Analytical expressions have been found for the output noise of a first order log-domain filter with stationary white noise injected at internal points.
- The calculations are in agreement with the experimental results.
- The proposed techniques can be extended to higher order filters.