

HW 2
E4215

solutions

Analog Filters.

Ans 2 (a)

Ckt 1

Ckt 2

Series RLC

Parallel RLC

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi \times 2.5 \times 10^4$$

$$C = 1 \text{ pF}$$

$$\Rightarrow L = 4 \mu\text{H}$$

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$$BW = \frac{R}{L}$$

$$BW = \frac{1}{RC}$$

$$\Rightarrow R = 19.7 \Omega$$

$$\Rightarrow R = 318.3 \Omega$$

(b) At 2.5 kHz i.e. at resonance frequency
 C, L cancel each other.

$$\therefore \text{for ckt 1 (series RLC)} \quad i_s = \frac{1V}{R} = \frac{1}{19.7} \text{ A}$$

for ckt 2 (|| RLC): $i_s = \frac{1V}{\infty} = 0 \text{ A}$
(C, L cancel each other and effectively
u have an open ckt at op)

$$(c) \text{ ckt 1} \quad BW = \frac{(R + R_s)}{L}$$

$$\Rightarrow \frac{(R + R_s)}{L} = 11 \text{ BW}_0 \quad (\text{where } BW_0 \text{ is } BW \text{ of } \text{parallel ckt } w/o R_s)$$

$$\frac{R+R_s}{L} = 1.1 \times BW_0 = 1.1 \left(\frac{R}{L} \right)$$

$$e) \frac{R_s}{L} = 0.1 \left(\frac{R}{L} \right)$$

$$R_s = \frac{R}{10} = 1.27 \Omega$$

ck 2 $BW = \frac{L}{C(R+R_s)} = 0.9 BW_0$

$$\frac{L}{C(R+R_s)} = 0.9 \frac{1}{CR} \Rightarrow R+R_s = \frac{10}{9} R$$

$$e) R_s = \frac{R}{9} = 35.3 \Omega$$

Ans 3 (a) $\omega_{PS} = \frac{1}{RL}$

(b) for a low pass (second order) ckt $BW \uparrow$ as $Q \uparrow$ moreover it is clear that we require maximally flat response, or butterworth response.

$$\therefore Q = \frac{1}{\sqrt{2}}$$

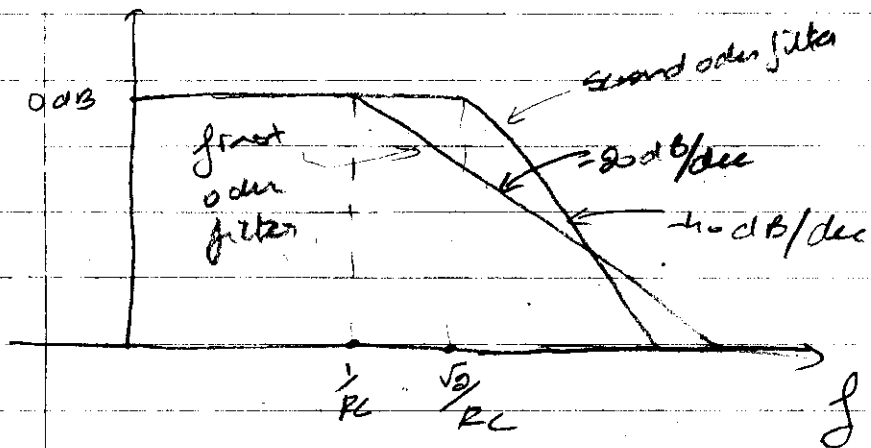
For series RLC $Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{\sqrt{2}}$

$$RL = \frac{R^2 C}{2}$$

For second order Butterworth filter

$$BW = \omega_0 = \frac{1}{\sqrt{LC}}$$

in terms of $R, C = \frac{1}{\sqrt{RC \frac{C}{R}}} = \frac{\sqrt{R}}{RC}$



Ans

Ckt 1

$$H(s) = \frac{1}{(1 + s/\omega_p)^2}$$

Ckt 2

$$V_o = A(V_i - V_o)$$

$$V_o + AV_o = AV_i$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{A}{1+A}$$

where $A = \frac{1}{(1 + s/\omega_p)^2}$

$$\therefore H(s) = \frac{1}{1 + \frac{1}{(1+s/\omega_p)^2}} = \frac{1}{1 + \left(\frac{1+s/\omega_p}{\omega_p}\right)^2}$$

(b)

CLT 1

$$H(s) = \frac{1}{1 + 2s/\omega_p + s^2/\omega_p^2}$$

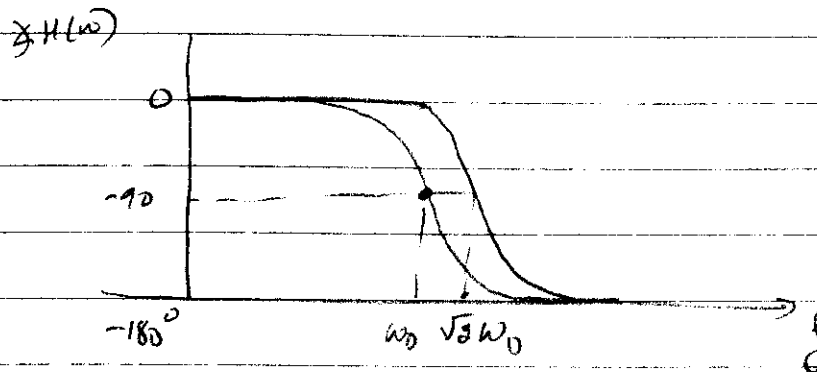
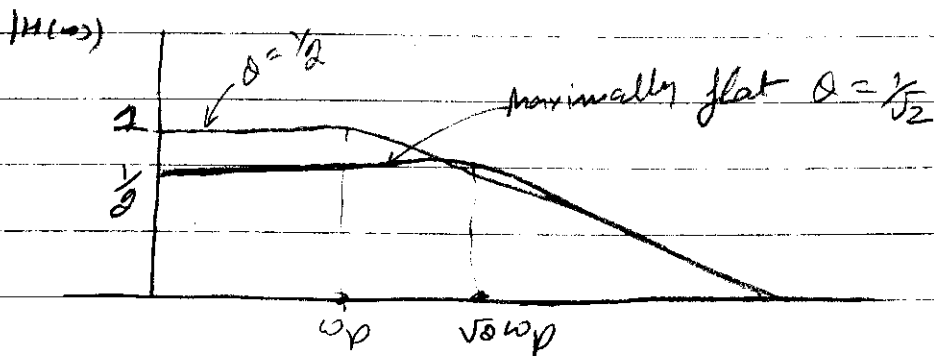
$\omega_0 = \omega_p \quad Q = \frac{1}{2}$

CLT 2

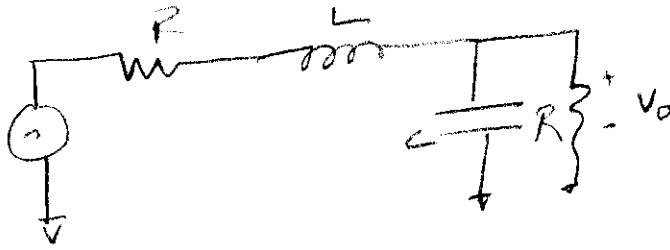
$$H(s) = \frac{1}{2 + 2s/\omega_p + s^2/\omega_p^2}$$

$$= \frac{1}{2} \frac{1}{1 + s/\omega_0 + \frac{s^2}{2\omega_0^2}}$$

$$\omega_0 = \sqrt{2} \omega_p \quad Q = \frac{1}{\sqrt{2}}$$



HW 9 Problem 1



$$\frac{V_o}{V_i} = \frac{R \parallel 1/sC}{(R \parallel 1/sC) + (R + sL)}$$

Voltage Divider

$$\approx \frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC} + R + sL} = \frac{R}{s^2 CLR + s(R^2 + L) + R}$$

$$\therefore \omega_0^2 = \frac{R}{CLR} = \frac{1}{CL} \quad (\text{constant})$$

$$BW = \frac{1}{CR} + \frac{R}{L}$$

$$Q = \frac{\omega_0}{BW}$$

$$\frac{\partial Q}{\partial R} = \omega_0 \frac{\partial}{\partial R} \left(\frac{1}{CR} + \frac{R}{L} \right)$$

$$= \omega_0 \left(-\frac{1}{CR^2} + \frac{1}{L} \right)$$

$$\frac{\partial Q}{\partial R} = 0$$

$$\Rightarrow R = \sqrt{\frac{L}{C}}$$

$$\therefore \text{Max } Q = \frac{1}{\sqrt{2}}$$

$$\& \omega_p = \text{constant} = \sqrt{\frac{1}{LC}}$$