HW 1

Analog Filters.
E4215
Prepared by
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Prob 1.

Fig 1(a)

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{-Z_2}{Z_1} \qquad Z_2 = R \parallel \frac{1}{sC}, \quad Z_1 = R$$

$$= \frac{-1}{1+sRC}$$

$$h(t) = \mathcal{L}^{-1} H(s) = \frac{-1}{RC} e^{-t/RC} \qquad \left[\mathcal{L}^{-1} \frac{1}{s+a} = e^{-at} \right]$$

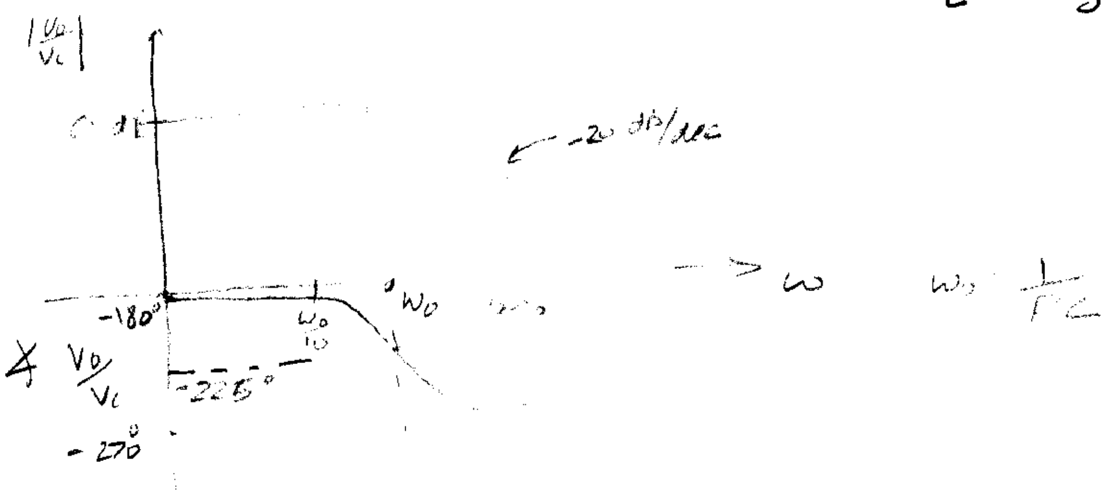


Fig 1(b)

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{-Z_2}{Z_1} \qquad Z_2 = 2R \parallel C/2, \quad Z_1 = 2R$$

$$= \frac{-1}{1+sRC}$$

H(s) is same for both ckts. $\therefore$ $h(t) = -\frac{1}{RC} e^{-t/RC}$
No diff. btw the ckts as far as Transfer function & impulse response are concerned

Prob 2

Fig 1(a) $\quad i_o(t) = -i_i(t)$

$$= -\frac{v_i(t)}{R}$$

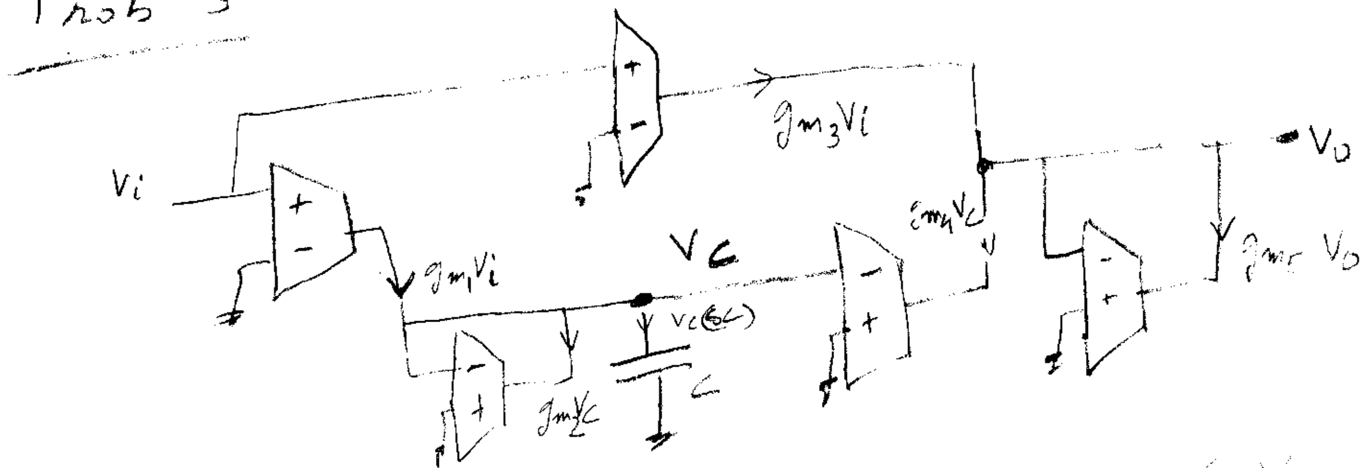
$$= -\frac{1}{R}\cos(t/RC)\ A$$

Fig 1(b) $\quad i_o(t) = -\frac{v_i(t)}{2R}$

$$= -\frac{1}{2R}\cos t/RC$$

The magnitude of input impedance for second ckt is double that of the first circuit

Prob 3



Apply Kirchhoff's current law at V_c & V_o

At V_c

$$g_{m1}V_i = g_{m2}V_c + V_c(sC) \qquad V_c = \frac{g_{m1}V_i}{g_{m2}+sC}$$

At V_o

$$g_{m3}V_i = g_{m4}V_c + g_{m5}V_o$$

Eliminating V_c

$$\left(g_{m3} - \frac{g_{m1}g_{m4}}{g_{m2}+sC}\right)V_i = g_{m5}V_o$$

$$\frac{V_o}{V_i} = \frac{g_{m3}(g_{m2}+sC) - g_{m1}g_{m4}}{g_{m5}(g_{m2}+sC)}$$

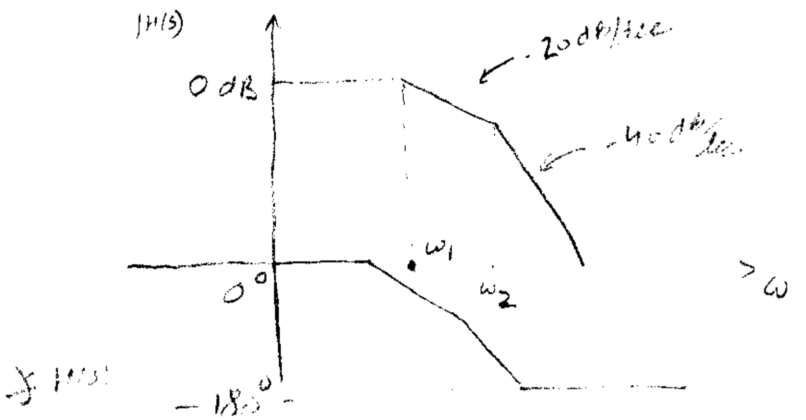
$$= \frac{g_{m3}g_{m2} - g_{m1}g_{m4}}{g_{m5}g_{m2}}\left(\frac{1 + \frac{sC}{\frac{g_{m3}g_{m2} - g_{m1}g_{m4}}{g_m}}}{1 + \frac{sC}{g_{m2}}}\right)$$

P4

Fig 3(a) Identifying the cascade of 2 inverting amplifiers.

$$H(s) = \frac{V_o}{V_i} = \frac{V_x}{V_i} \cdot \frac{V_o}{V_x}$$

$$= \frac{1}{(1+sR_1C_1)(1+sR_2C_2)}$$ (Refer Prob 1(a))



$\omega_1 = 1/R_1C_1$

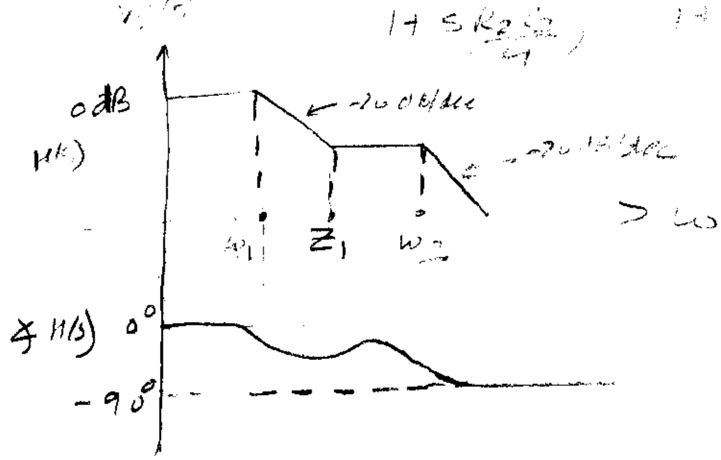
$\omega_2 = 4\omega_1 = 1/R_2C_2$

Fig 3(b)

$$H(s) = \frac{V_x}{V_i} \cdot \frac{V_o}{V_x}$$

$$\frac{V_o(s)}{V_x(s)} = \frac{\dfrac{R_2}{1+s\frac{R_2C_2}{4}}}{\dfrac{R_2}{1+sR_2C_2}} = \frac{1+sR_2C_2}{1+s\frac{R_2C_2}{4}}$$

$$\therefore\ H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1+sR_2C_2}{1+s\frac{R_2C_2}{4}} \times \frac{1}{1+sR_1C_1}$$



$\omega_1 = \frac{1}{R_1C_1}$

$Z_1 = \frac{1}{R_2C_2}$

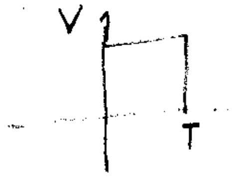
$\omega_2 = \frac{4}{R_2C_2}$

Prob 5

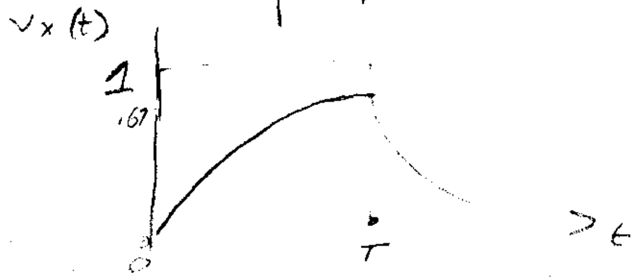
$$\frac{V_x(s)}{V_i(s)} = \frac{1}{1 + sR_1C_1}$$

These are lowpass filter characteristics

$\therefore$ If $V_i(t)$



$T = R_1C_1$



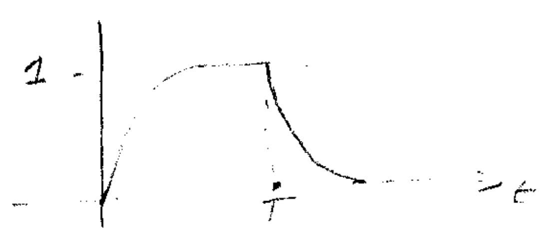
Time constant of exponential increase & decay will be $\tau = R_1C_1$

$$\frac{V_o(s)}{V_i(s)} = \frac{1 + sR_2C_2}{1 + s\left(\frac{R_2C_2}{4}\right)} \times \frac{1}{1 + sR_1C_1}$$

$$= \frac{1}{1 + s\left(\frac{R_2C_2}{4}\right)} \quad \text{if } R_1C_1 = R_2C_2$$

Time constant of filter $= \frac{R_1C_1}{4} < T$

$\therefore$ $V_o(t) \cong$



Refer Appendix F ...

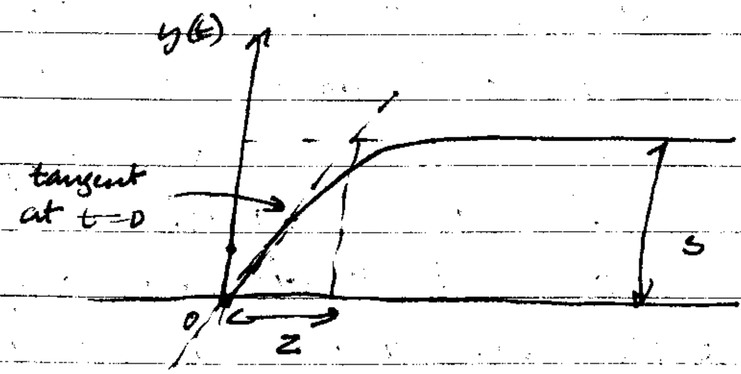
Prob 5

$$\frac{V_x(s)}{V_i(s)} = \frac{1}{1+sR_1C_1}$$

These are lowpass filter characteristics.

In response to a step signal of height S, a low pass STC produces a waveform as shown below

$$y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau} \qquad \tau = RC$$



where $Y_\infty \equiv$ final value or value at $t=\infty$

$Y_{0+} \equiv$ value immediately after $t=0^+$

In our case $Y_\infty = S$ and $Y_{0+} = 0$

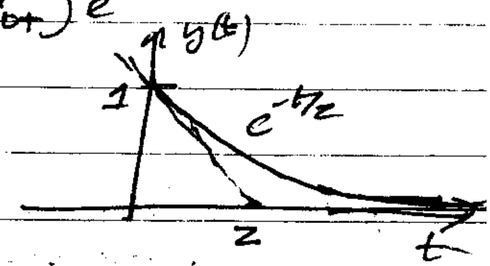
$$\therefore \quad y(t) = S(1-e^{-t/\tau})$$

Similarly in response to a falling step (on a -ve step)

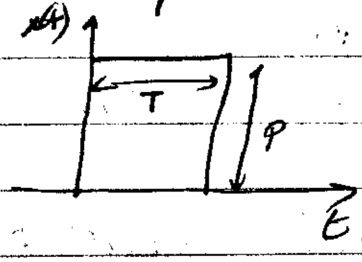
$$y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$$

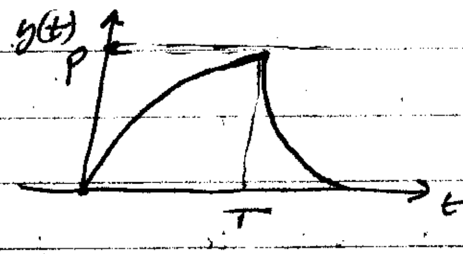
$Y_\infty = 0 \qquad Y_{0+} = 1$

$$\therefore \quad y(t) = e^{-t/\tau}$$



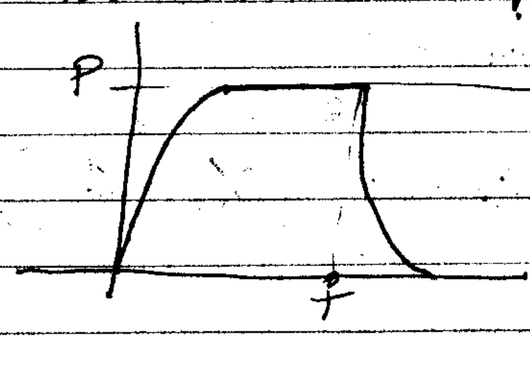
∴ A low pass STC ~~as~~ response to a pulse will be as shown below





The LP circuit does not respond to the step change at the leading edge of the pulse, rather the output starts to rise exponentially towards final value of P. The exponential rise is however stopped at time $t = T$, at the trailing edge of the pulse. The output will then respond by starting an exponential decay toward final value of the input which is zero.

In well designed system, in which pulses are to be transmitted (ex. digital systems) distortion due to low pass effects is kept low by arranging τ to be much smaller than width T. ~~This~~ this case response is as shown.



Refer Sedra & Smith
Appendix E

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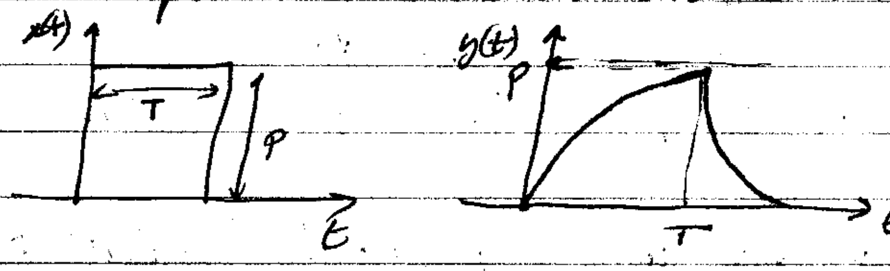

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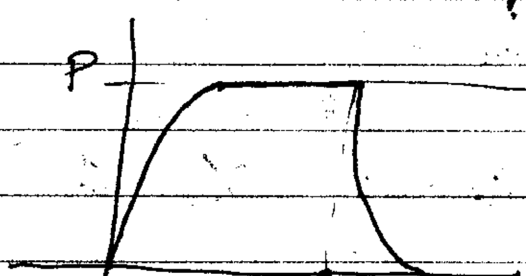

Refer - Sedra & Smith
Appendix F