E4215: Analog Filter Synthesis and Design
Frequently used numerical approximations

Nagendra Krishnapura (nkrishnapura@mltc.com)

12 Feb. 2003

While calculating the effects of nonidealities of real components on the transfer function of a filter, following aproximations may be used to simplify the expressions assuming that the deviations are small.

1 For small quantities

$$
\begin{aligned}
(1+x)^\alpha &\approx 1+\alpha x \\
& |x| \ll 1,\ |\alpha x| \ll 1 \\
& 1.1^2 = 1.21;\ (1+0.1)^2 \approx 1.2;\ \text{-0.83\% error} \\
\frac{1}{1+z} &\approx 1-z \\
& |y| \ll 1,\ \text{use } \alpha=-1 \text{ in previous expression} \\
& \frac{1}{1.1} = 0.90909;\ \frac{1}{1+0.1} \approx 0.9;\ \text{-1\% error} \\
(1+x)(1+y) &\approx 1+x+y \\
& |x| \ll 1,\ |y| \ll 1 \text{ means } xy \text{ is negligible compared to } 1, x, y
\end{aligned}
$$

The last expression indicates that if two factors contribute to 1 % deviation, say in the pole frequency, the combined effect can be expected to be 2 % in the worst case. In general, for small deviations, the deviation due to each factor can be calculated separately and added up.

$$
\begin{aligned}
\frac{1+x}{1+z} &\approx 1+x-z \\
& |x| \ll 1,\ |z| \ll 1 \text{ combine last two; } y=-z
\end{aligned}
$$

This suggests that if two factors cause equal and opposite deviations, there *may*[1] be an opportunity to cancel the deviations by combining the two factors in the same circuit.

$$
\begin{aligned}
\exp(x) &\approx 1+x \\
& |x| \ll 1
\end{aligned}
$$

[1]This sort of cancellation technique should always be investigated for reliability

$$\ln(1+x) \approx x$$

2 Quadratic equation

$ax^2 + bx + c = 0$ has two roots

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which are cumbersome to calculate in general. Simple approximations are possible when one of the roots is much larger than the other ($|x_1| \gg |x_2|$). In $f(x) = ax^2 + bx + c$, the first two terms dominate when x is "large" and the last two, when x is small. So, to calculate the larger root x_1,

$$\begin{aligned} ax_1^2 + bx_1 &\approx 0 \\ x_1 &\approx -\frac{b}{a},\ 0 \text{ (0 not permissible: } x_1 \text{ is assumed to be large)} \end{aligned}$$

and the smaller root x_2

$$\begin{aligned} bx_2 + c &\approx 0 \\ x_2 &\approx -\frac{c}{b} \end{aligned}$$

Hence the algorithm:

- Given a quadratic equation $ax^2 + bx + c = 0$, calculate $x_1 = -b/a$ and $x_2 = -c/b$.
- Verify if $|x_1| \gg |x_2|$. If true, they are the roots. If not, recalculate exactly.

This approximation clearly doesn't work when the roots are complex because, then, $|x_1| = |x_2|$.

e.g. $x^2 + 11x + 10$ yields $-10, -1$ exactly and $-11,\ -10/11$ using the approximation, which are correct to 10 %.